Frequency Response of Tires Using the Point Contact Theory

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The theoretical frequency response of a rolling tire is obtained here for angular oscillations about a vertical axis through the axle in the wheel plane. The point contact theory, also known as the DeCarbon or Moreland theory, is used to describe the side (cornering) force and the torsional (self-aligning) moment as a function of the oscillating frequency. These results are compared to previously published experimental results and found to show reasonable correlation especially when compared to theoretical results obtained using the von Schlippe or stretched string theory. It is noted, both from experimental and theoretical results, that the frequency response is considerably influenced by rolling speed which is contrary to implications given in previous papers. It is also pointed out that an upper limit of the forcing frequency exists above which the empirical data is not likely to represent natural shimmy oscillations. The results indicate that the point contact theory is adequate for use in shimmy analysis over both the tire yaw and structural-torsion shimmy modes of vibration if appropriate tire parameters are used.

Nomenclature

b = half width of tire footprint or contact area
c = tire yaw coefficient
c_L = tire lateral damping coefficient
c_1 = tire time constant
D = tire diameter
D_{drum} = drum diameter
d = effective tire width
F = magnitude of dimensionless, complex lateral force amplitude
$\bar{F}$ = dimensionless complex lateral force
F_t = lateral force acting on wheel
F_z = vertical force on wheel
f = oscillation frequency, cps
h = half length of tire footprint or contact area
i = imaginary unit $(-1)^{1/2}$
k_1 = lateral tire stiffness
M = magnitude of dimensionless, complex tire moment amplitude
$\bar{M}$ = dimensionless complex tire moment
M_t = moment acting on wheel
M^* = moment about vertical axis due to finite tire width
p, p_r = tire inflation pressure, rated pressure
R, R_r = tire radius, rolling radius
$\bar{R}$ = ratio of footprint length to quarter path wave length
s = complex frequency $(i\bar{\omega})$
V = forward rolling speed
x, y = lateral and forward wheel coordinates
Δ = lateral deformation of tire
δ = vertical deflection of tire
$\dot{\theta}, \dot{\theta}_0$ = angular velocity of tire for slip and no slip
λ = tire path wave length
μ_1, μ_d = tire torsional stiffness, and damping coefficient
ϕ_F, ϕ_M = phase angle of lateral force and moment
ψ, ψ_0 = yaw angle of wheel and its amplitude
ψ_t = yaw angle of tire footprint relative to wheel
ω = radial oscillation frequency, rps
$(\dot{\ })$ = derivative with respect to time
$(\ )'$ = derivative with respect to y coordinate

I. Introduction

An important aspect in the design of aircraft and automotive wheel systems is the vibration of the wheel about its vertical axis commonly referred to as shimmy. If the designer is to perform a realistic shimmy analysis he must, of course, use valid mathematical relationships describing the force-deflection characteristics of the tire. Several theoretical models of tires have been proposed in the literature, and the most realistic and useful of these are the stretched theory[1] and the point contact theory.[1,5] Although the stretched string theory has been proven valid in many respects, it leads to a delay-differential equation which can cause difficulties in the solution of shimmy problems both in the direct integration of the equations of motion and in the application of stability criteria. It has been shown[1,5] that the point contact theory will provide valid results for low-frequency shimmy oscillations while offering computational simplicity over the stretched string theory. It still remains, however, to show the degree of validity of the point contact theory for the higher shimmy frequencies in the structural torsion shimmy range.[5] It is the purpose of this paper to show that the point contact theory described in previous papers can provide a valid mathematical representation of the tire over-all possible shimmy frequencies. In the following the frequency response equations for the point contact theory is formulated and compared to the theoretical results using some published experimental results over the range of realistic shimmy frequencies.

It has been recently noted by the author that an empirical formulation of the tire equations has been suggested by Rogers and Brewer[8] using Bode plots of certain experimental data. Formulation of tire equations by synthesis from experimental data can lead to very good results for any particular set of experimental data depending upon the order of the differential equations assumed. However, the necessary coefficients obtained in this manner, without regard to the actual mechanics of the system, do not lend themselves readily to extrapolations from the conditions of the experimental data from which they were determined. It may be observed by the reader that the synthesis technique proposed in Ref. 8 (contrary to statements made there) may be applied to the more rational theoretical tire equations of Refs. 2 and 3 as well as those presented in the following pages.

II. Experimental Frequency Response Data

Consider a pneumatic tire mounted upon a wheel which has its center plane vertical and axis of rotation perpendicular to the direction of rolling as indicated in Fig. 1. In this state the wheel experiences only a drag force resulting from the resistance of the tire carcass to its vertical deflection caused by imposing a vertical load on the axle. In what follows the drag force is assumed negligible in comparison to the other forces which occur when the wheel plane is altered in direction or lateral position. If the wheel plane is rotated about a vertical axis through its axle an angle ψ, a side force F_t and a

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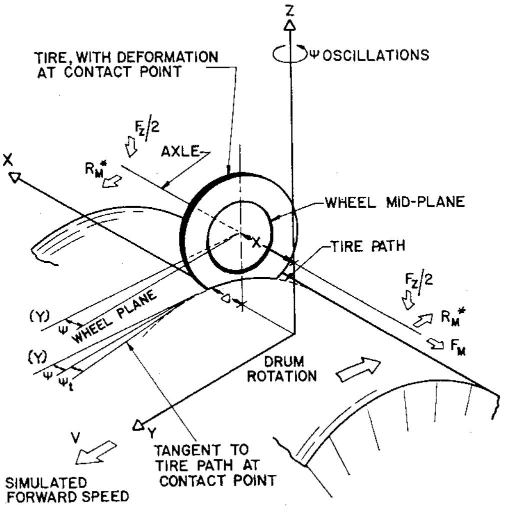

Fig. 1 Schematic drawing of tire frequency response test set up.

moment M_t are experienced by the wheel. Also, if the wheel plane retains its direction during rolling but is displaced at a rate $\dot{x}$ to the side a force and a moment will again be experienced. The tire tests considered here were performed on wheels mounted over a rotating drum to simulate forward rolling as indicated in Fig. 1.

Frequency response data is generally obtained for harmonic angular motions of the wheel plane such as: $\psi = \psi_0 \cos\omega t$, where the amplitude ψ_0 is fixed and the resulting lateral (or side) force F_t and moment M_t are determined by gages fixed to the axle. The measured force and moment values should be corrected for the effects of the wheel and tire inertia as: $F_t = F_m - m\ddot{x}$ and $M_t = M_m - I\ddot{\psi}$, where F_m and M_m are the measured values of the side force and the moment. In the experimental data available only the ψ motion is produced while the x motion is restrained so that $x \equiv \dot{x} \equiv \ddot{x} \equiv 0$.

The data is presented in typical frequency response fashion where the output F_t, M_t are ratios to their stationary, steady state ($\omega = 0$) values. For a given frequency ω these output ratios will have a magnitude and a phase angle ϕ relative to the input ψ. The magnitudes are represented by: $F = |F_t/\psi|/|F_t/\psi|_s$; $M = |M_t/\psi|/|M_t/\psi|_s$ where the subscript s refers to stationary, steady-state conditions and the phase angles are represented by: ϕ_F and ϕ_M. These ratios when plotted against the input frequency ω are the common attenuation and phase plots. However, rather than plotting against the frequency ω the method used in previous papers[2-4,6,8] will be used here also. The reduced frequency $\bar{\omega}$ is used where $\bar{\omega} = \omega/V$ and where V is the rolling speed. The data was previously published using $\bar{\omega}$ in place of ω probably because it was assumed that the results would be independent of rolling speed V. It will be noted, however, that this is generally not true. Some of the previous papers also use a somewhat arbitrary dimensionless parameter $\bar{\omega}D$ where D is the tire diameter. Actually a more characteristic dimensionless parameter would be $2\bar{\omega}h$ where $2h$ is the length of the tire contact or "footprint."

The frequency response data discussed here was obtained over a large range of reduced frequencies for a few automotive type tires. It can be noted that areas where the experimental data begins to deviate substantially from the linear theories are regions where significant tire contact slippage is very likely taking place. In this case one cannot reasonably expect the linear theories based on an adhesive contact region to be reliable. It will be shown in the following that, in several cases considered, shimmy did not occur above a value of $\bar{\omega}$ for which the static footprint length is greater than one-quarter of the tire track wave length.

Realistic Reduced Frequency Range for the Occurrence of Shimmy

To determine values of the reduced frequency $\bar{\omega}$ for which shimmy might occur the following hypotheses are considered: 1) Shimmy is caused by a tire contact stress condition which develops while the tire is principally in a state of pure adhesive rolling. Significant sliding in the contact region becomes prevalent only when the shimmy oscillations have grown sufficiently large in frequency or in amplitude for the tire-ground lateral adhesive forces to be overcome. 2) When the static tire footprint length is greater than one quarter of tire path wavelength, the tire can be considered as principally in a state of sliding, which is not a natural shimmy condition. By tire path wave length we mean the wave length of the trace of the tire upon the rolling surface.

If f is the tire path frequency in cycles per second ($f = \omega/2\pi$) then the path wavelength λ is $\lambda = V/f$ and from the definition of $\bar{\omega}$ the wavelength is $\lambda = 2\pi/\bar{\omega}$. Let $\tilde{R}$ be the ratio of the footprint contact length ($2h$) to the quarter wavelength $\lambda/4$:

$$\tilde{R} = 2h/(\lambda/4) = 4h\bar{\omega}/\pi \tag{1}$$

Fig. 2 shows the range of natural shimmy frequencies experienced during shimmy testing of various aircraft landing gear systems and models. Some of the data has been taken directly from aircraft landing gear which experienced shimmy during aircraft taxi tests. Other data is from aircraft landing gears and small model tires mounted above rotating drums. The data indicates a definite range of frequencies within which natural shimmy motion occurs. It appears that shimmy for typical aircraft systems occurs only for $\tilde{R}$ ratios less than one, a conclusion which may also be drawn from hypotheses (1) and (2) which were previously mentioned.

From this collaboration of experimental data and intuition the following criterion for determining an upper limit on the reduced frequencies for which natural shimmy oscillations are capable of being excited is proposed: The footprint ratio $\tilde{R} = 1$ is a practical upper limit for the occurrence of shimmy. At combinations of $\bar{\omega}$ and h which produce $\tilde{R} > 1$ it is improbable that sufficient nonsliding footprint contact area is developed so that natural shimmy oscillations can be maintained. Using Eq. (1) and the limit $\tilde{R} = 1$ it is observed that the reduced frequency range over which the occurrence of shimmy is possible may be determined from the inequality

$$h\bar{\omega} < 0.787 \tag{2}$$

It can be noted that some of the experimental forced frequency response data discussed herein was obtained well beyond this limit.

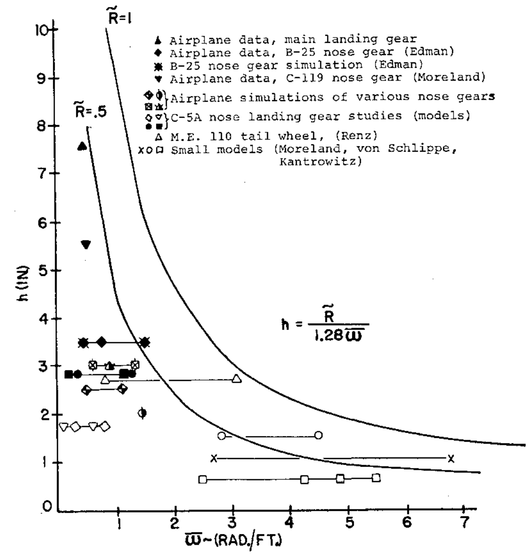

Fig. 2 Comparison of reduced frequencies experienced during shimmy testing of aircraft tires.

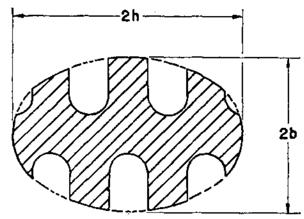

Fig. 3 Sketch of contact area of block pattern tread tire used to obtain Pacejka's frequency response data.

The most complete set of data on the frequency response of tires that the author was able to find published is presented by Pacejka.[3] A careful search of this thesis yields the information necessary for the correlation presented in the following. Pacejka's tests† were made by pressing the tire onto a rotating steel drum. The tire force and reactions were measured at the axle and were corrected for tire and wheel inertial effects. In the thesis, data is presented for two truck or tractor-type tires with considerably different tread design. Only the tread design referred to as "block pattern" is presented here as the larger portion of the available data is for this tire. A sketch of the block pattern tread footprint is shown in Fig. 3.

The following information, presented in the thesis in kms units has been converted for use in Table 1:

To obtain data over a range of values of $\bar{\omega}$ Pacejka held the frequency ω constant while varying the speed V. In presenting data in this manner it is important to note that the resulting response curves are rolling speed response curves at constant frequency rather than true frequency response curves. If the tire properties are independent of speed, the particular frequency ω used during the tests will be unimportant and all response curves will coincide. However, the data shows a very definite effect due to rolling speed V. If the rolling speed of the tire is considered to be a property of the physical state of the tire then it would seem more appropriate to hold this quantity constant while changing the driving frequency ω in order to obtain the frequency response. Some of Pacejka's data are replotted herein using constant velocity as a parameter.

Four frequencies were used ($f = 1$, 2.16, 4 and 8 cps) so that a reasonable cross plot of the original complex plane data can be made to obtain the frequency response curves shown in Fig. 4 a–c. The original data appears on p. 173, Fig. 11 of Ref. 3. The data appear to be quite good and exhibits reasonable trends. As in all tire data, however significant scatter can be expected due to the very complexity of the actual tire mechanics. For instance varying the amplitude of oscillation or the type of tire tread can change the results considerably as is exhibited by other data in Refs. 3 and 7.

The reduced frequency limit of Eq. (2) can be found from the tire data given above as $\bar{\omega} < 0.787/0.378 = 2.08$ rad/ft. It can be observed in Fig. 4 a–c that reversals in trend of the output quantities ϕ_F, M, and ϕ_M take place in the range $1 < \bar{\omega} < 2$ which indicates that natural shimmy oscillations are not likely to occur after this reversal takes place since the

Table 1 Tire: automotive (heavy duty) 9.00 × 16

$D = 1$ m $= 3.28$ ft	;	$D_{drum} = 8.2$ ft	
$F_z{}^* = 6960$ n $= 1565$ lb	;	$\Psi_0 = \pm 0.75°$	
$p = 25$ psi	;	$p_r = 55$ psi	
$h = 0.115$ m $= 0.378$ ft	;	$b = 0.0925$ m $= 0.303$ ft	
$c = 0.75 \times 10^{-4}$ rad/lb	;	$k_1 = 1.133 \times 10^4$ lb/ft	
$\mu_1 = 2.88 \times 10^3$ ft lb/rad			

† Pacejka is not clear as to the value of vertical load for these tests. He gives a value of 10,000 for another series of tests in which the contact half-length is $h = 0.138$m. The contact half-length for this data is $h = 0.115$m and assuming vertical load varies as the square of the contact length, the load for this case is: $10{,}000 \times (0.115/0.138)^2 = 6960$ N.

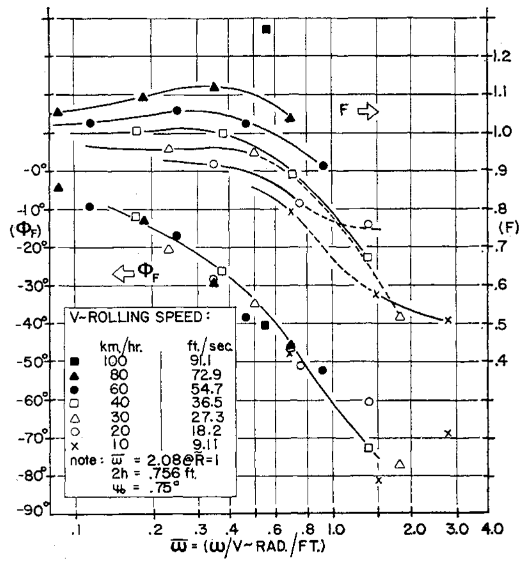

Fig. 4a From Pacejka's experimental data. Normalized lateral tire force amplitude and phase angle as a function of the reduced frequency for various rolling speeds.

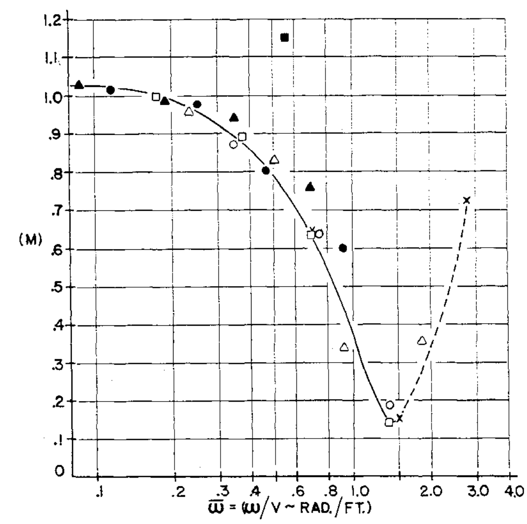

Fig. 4b From Pacejka's experimental data. Normalized tire moment amplitude as a function of the reduced frequency for various rolling speeds.

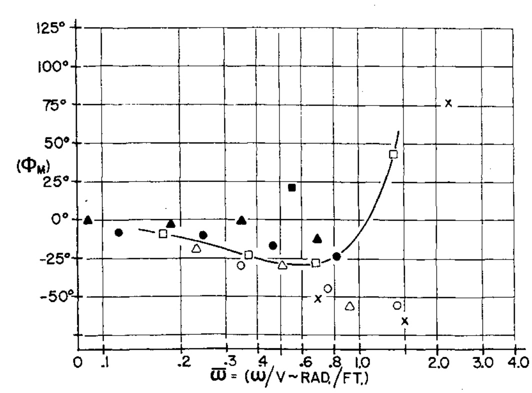

Fig. 4c From Pacejka's experimental data. Tire moment phase angle as a function of reduced frequency for various rolling speeds.

tire is being forced into a state of considerable footprint slippage. The parameter in Figs. 4 a–c is rolling speed V and it can be seen that considerable effects due to rolling speed are present. For this limited quantity of data the most significant speed effects appear in the lateral force response F.

Although this is an automotive type tire, the test inflation pressure is only 25 psi compared to the 55 psi rated normal pressure. Therefore, the contact length to the tire diameter ratio (0.23) more closely represents some typical aircraft tires than it does a standard automotive tire (about 0.10).

Saito[4] presents some frequency response data for a pneumatic "model tire" mounted on a rotating steel drum. Some numerical information on the test equipment is given however no data is available on the tire parameters (stiffness, etc.) beyond the tire diameter (158 mm). The vertical load was 3.5 kg but without the vertical tire stiffness values for the footprint length $2h$ cannot be determined so that the reduced frequency limit of Eq. (2) cannot be established. It is likely that the data obtained in the higher reduced frequency range is above this limit however due to the appearance in a sudden shift of the tire moment phase angle ϕ_M from about $-65°$ to $+100°$ at $\bar{\omega} \doteq 5.0$. Data is obtained to about $\bar{\omega} = 9.5$ rad/ft.

A significant factor is noted in the absence of data on the rolling speed (V) and true frequency ω (rad/sec) which implies that the author considered the effects of rolling speed to be insignificant and which, as shown in Pacejka's data, is a serious oversight. As a matter of fact by neglecting V as a parameter and plotting the frequency response curves in this manner one can be mislead into labeling the reduced frequency at the point where the tire moment phase angle ϕ_M shifts from negative to positive as the point of "kinematic shimmy." This was done by Saito as well as others. The term kinematic shimmy coined by A. Kantrowitz some years ago refers to wheel oscillations at exceptionally low speed V where inertial effects are essentially nonexistent. However, as noted in Fig. 4 the condition of the ϕ_M "shift" does also occur at the high rolling speeds and consequently high shimmy frequencies ω so that the shimmy, if it could occur naturally at this frequency, would hardly be of the "kinematic" mode. It may be concluded that this condition, rather than indicating a point of kinematic shimmy, indicates the frequency at which significant slippage occurs in the footprint and consequently where the ratio $\tilde{R}$ is approaching the limit ($\tilde{R} = 1$).

J. L. Ginn[6] et al. also obtained data on a 28 in. diam automotive tire mounted so that it rolled upon a 120 in. diam steel drum. The tire contact length, although not explicitly stated, can be calculated from other information ($2h = 0.75$ ft). The reduced frequency limit from Eq. (2) gives $\bar{\omega} < 2.1$ rad/ft. The data shows the tire moment phase ϕ_M shifts from about $-65°$ to $+70°$ at $1.5 < \bar{\omega} < 2.15$ rad/ft. The tests were run at two oscillation frequencies ($f = 1$ cps, 4 cps) and the rolling speed was varied at each frequency in order to obtain a range of reduced frequencies from $0.06 < \bar{\omega} < 7$ rad/ft. With this data the effect of speed V can be determined to some degree. The effect was not large in this case but could be observed on the force phase angle ϕ_F and the moment amplitude M. It is possible that the rather large oscillation amplitudes used in these tests ($\pm 4°$) led to excessive contact area slippage and masked some of the speed effects.

III. Theoretical Frequency Response

It is possible to determine the frequency response of tires by theoretical analysis over the natural shimmy frequency range with good accuracy. Segel,[2] Saito,[4] and Pacejka[3] have performed analyses using the stretched elastic beam and stretched string concepts for the tire. The frequency response analysis using the stretched string and beam theories will not be presented here as they are covered in the cited references.

The equations for the point contact theory as presented in Ref. 1 are recast here into the frequency response form with the path distance variable y replacing time t as the independent variable. If primes denote derivatives with respect to y then the lateral or side force acting on the wheel is

$$F_t = (\psi_t + c_1 V \psi_t')/c \tag{3}$$

where ψ_t is the angle between the wheel plane and the tangent to the tire path as shown in Fig. 1 (also see Ref. 1). This angle represents the amount of twist of the tire footprint with respect to the wheel plane. The parameters c_1 and c are the tire time constant and the tire yaw coefficient while V is the rolling speed. The side force is also related to the lateral deflection Δ of the tire footprint center relative to the wheel plane as

$$F_t = k_1 \Delta + c_L V \Delta' \tag{4}$$

where k_1 is the lateral tire spring constant and c_L the lateral tire damping coefficient.

The tire footprint yaw angle ψ_t also leads to a torsional moment M_t as indicated in Fig. 1. which may roughly be represented by:

$$M_t = \mu_1 \psi_t + \mu_d V \psi_t' + M^* \tag{5}$$

where μ_1 is the torsional spring constant of the tire and μ_d the torsional damping coefficient. The moment M^* is due to the finite width of the tire and its resistance to the steering rate $\dot{\psi}$. When the tire direction is changed due to the rate $\dot{\psi}$ one side of the tire contact region is forced to operate at a greater rolling rate while the other rolls at a slower rate thereby inducing a tractive force on one side and a braking force on the other. This effect then leads to a torsional moment about the vertical axis in the wheel plane which retards the angular rate $\dot{\psi}$. For the sake of brevity a complete derivation of the moment will not be given here however the following explanation is offered.

The moment M^* can be shown to be roughly equivalent to

$$M^* = dkR_r(\phi' - d\psi'/R_r)/2 \tag{6}$$

where d is the "effective" width of the tire, (approximately half the tire width), k is the coefficient of slip defined by the equation $F_g = kS$ where F_g is the tractive or braking force acting on each side of the tire in the ground contact region and S the slip where $S = (\dot{\theta}_0 - \dot{\theta})/\dot{\theta}_0$. The tire rotational rate is denoted by $\dot{\theta}$ while $\dot{\theta}_0$ is the rotational rate when no braking or traction is present. The constant (R_r) is the tire rolling radius[7] ($R_r \doteq R - \delta/3$) where δ is the vertical tire deflection. The tire is approximated by two disks separated by a "torsion rod" of length d and stiffness K_ϕ. One disk turns relative to the other an angular amount ϕ so that a torque $T = K_\phi \phi$ is experienced by each disk (in opposite directions). When this torque, the ground force F_g and the slip S are used in the equations of equilibrium for the two disks and if inertial effects ($I\ddot{\phi}$) and damping effects ($C_\phi \dot{\phi}$) are neglected Eq. (6) may be derived along with the following expression:

$$\phi' = d\psi'/R_r + 2K_\phi \phi/[kR_r(R - \delta)] \tag{7}$$

from which ϕ may be determined in terms of ψ.

Another expression which is necessary in tire rolling problems is the constraint of rolling without slipping. For the point contact theory this equation is simply

$$\Delta' + \psi_t = -(\psi + x') \tag{8}$$

Eqs. (3)–(8) can now be combined so that the "output" F_t, M_t can be written in terms of the "input" ψ and x', the variables ψ_t, Δ, and ϕ being eliminated.

Since the final results will be written as normalized ratios to stationary, steady-state value it is useful to consider the preceding equations at this condition ($\psi_t' = \Delta' = \phi' \equiv 0$)

$$(F_t)_s = (\psi_t)_s/c;\ (M_t)_s = \mu_1(\psi_t)_s$$
$$(\Delta)_s = (\psi_t)_s/(k_1 c);\ (\psi_t)_s = -(\psi)_s;\ (M^*)_s = 0$$

where we also use the condition $x' \equiv 0$ in this paper.

To find the frequency response to the input $\psi = \psi_0 \exp(i\omega t)$ we note that as $y = Vt$ where t is the time, then $\omega t = \bar{\omega} y$ and letting $s = i\bar{\omega}$ we substitute into Eqs. (3–8) above: $\psi = \psi_0 \exp(sy)$, $\Delta = \Delta_0 \exp(sy)$ and eliminate ψ_t, Δ, and ϕ leaving

$$\bar{F} = (F_t/\psi)/(F_t/\psi)_s = (as^2 + bs + k_1)/(As^2 + Bs + k_1) \quad (9)$$

$$\bar{M} = (M_t/\psi)/(M_t/\psi)_s = M' + \bar{M}^* \quad (10a)$$

where

$$M' = (\bar{a}s^2 + \bar{b}s + k_1)/(As^2 + Bs + k_1) \quad (10b)$$

$$\bar{M}^* = [(\bar{A}d/(\mu_1 R_r)]s/(1 - \bar{c}s) \quad (10c)$$

where the coefficients in Eqs. (9) and (10) are

$$a = c_1 c_L V^2; \quad b = (c_L + c_1 k_1)V; \quad A = c_1 V/c;$$
$$B = c_L V + 1/c; \quad \bar{a} = c_L \mu_d V^2/\mu_1; \quad \bar{b} = (c_L + \mu_d k_1 V/\mu_1)V;$$
$$\bar{A} = dkR_r/2; \quad \bar{C} = kR_r(R - \delta)/(2K_\phi)$$

With $s = i\bar{\omega}$ we have the complex frequency response equations for $\bar{F}$ and $\bar{M}$. The magnitude of these complex quantities will be denoted by F and M while the arguments will be the corresponding phase angles of $\bar{F}$ and $\bar{M}$ denoted by ϕ_F and ϕ_M. Separating $\bar{F}$ into its real and imaginary parts we find by conventional methods:

$$F = \{[\text{real}(\bar{F})]^2 + [\text{imaginary}(\bar{F})]^2\}^{1/2}$$
$$\phi_F = \tan^{-1}[\text{imag.}(\bar{F})/\text{real}(\bar{F})]$$

similarly for M. Since these operations are quite well-known and since the resulting expressions are algebraically quite complicated and do not give significant information over Eqs. (9) and (10), they will not be presented here. The expressions for F, M, ϕ_F, ϕ_M are functions of $\bar{\omega}$ and V and can be compared directly with the experimental data presented in this paper.

The M^* moment of Eq. (6) was not used in previous papers concerned with the point contact theory; however, Pacejka[3] developed a similar term and found it necessary for correlating the stretched string theory with experimental results for the higher frequency oscillations. The M^* term is also required to provide valid results with the point contact theory at higher frequencies near $\bar{R} = 1$. The effect of M^* on the moment M_t becomes more prominent as $\bar{\omega}$ increases and may best be investigated by considerations of M' and $\bar{M}^*$ drawn in the complex $\bar{M}$ plane; however, these details are beyond the scope of the present paper and we will now pass to the more pressing problem of establishing the validity of these equations by experimental-theoretical correlation.

IV. Experimental-Theoretical Correlation

The validity of the frequency response Eqs. (9) and (10) derived in Sec. III is examined here by comparing solutions of those equations with the experimental data of Sec. II. As a first estimate of the tire parameters c, k_1, μ_1, and R the values from Sec. II of this paper are used. However, to solve for the force and moment response from the Eqs. (9) and (10) values of c_1, c_L, μ_d, δ, d, k and K_ϕ are also needed. Data on these quantities is virtually nonexistent. Ref. (7) gives an estimate of the deflection δ and a crude check on c, k_1, and μ_1. From Refs. 5 and 7 it is also noted that considerable variation in all tire parameters can be expected depending upon many factors such as rolling surface condition, tire tread condition, and rolling speed V. Because of the lack of experimental data the resulting analysis becomes somewhat academic, however, the qualitative nature of the resulting trends remains of considerable importance and interest to analysts and designers concerned with the shimmy problem.

The effective tire width d is chosen to be about 75% of the true tire width $2b$. A procedure was developed for determining values of c_1, c_L, μ_d, k and K_ϕ which would match the data presented in Figs. 4 a–c for a rolling speed of $V = 36.5$ fps (40 km/hr) within reasonable limits for these quantities. Of course, these reasonable limits are again only estimates from previous data on tires, mostly of the aircraft type. Absolutely no reliable data exists; as far as the author is aware on the values of c_L, μ_d, and K_ϕ under dynamic or rolling conditions.

The following values were found as reasonable estimates of the tire parameters for $V = 36.5$ fps:

$c = 0.6 \times 10^{-4}$ rad/lb,	$k_1 = 1.0 \times 10^4$ lb/ft
$\mu_1 = 1 \times 10^3$ ft lb/rad ,	$c_1 = 0.019$ sec
$c_L = 20$ lb sec/ft ,	$\mu_d = 18$ ft lb sec
$R = 1.64$ ft ,	$\delta = 0.045$ ft
$d = 0.45$ ft ,	$k = 4.7 \times 10^3$ lb
$K_\phi = 40 \times 10^3$ ft lb/rad,	$V = 36.5$ ft/sec

When these parameters are used in Eqs. (9) and (10) the force and moment response are obtained over a range of reduced frequencies $\bar{\omega}$. This response is plotted in Figs. 5 a–c.

For rolling speeds other than $V = 36.5$ fps values of the tire parameters should be determined experimentally and then an exact correlation of the frequency response using the point

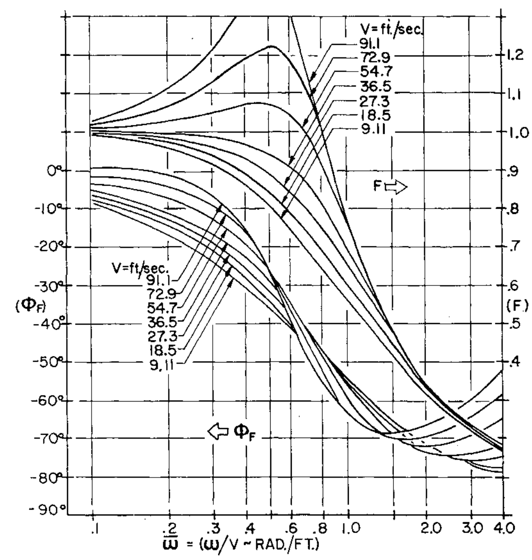

Fig. 5a Theoretical, normalized lateral tire force amplitude and phase angle as a function of the reduced frequency for various rolling speeds.

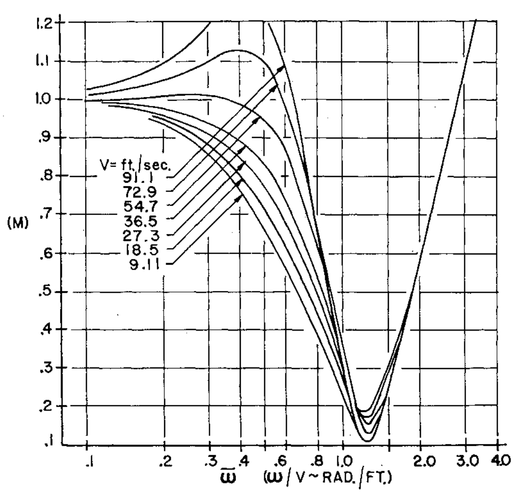

Fig. 5b Theoretical, normalized tire moment amplitude as a function of the reduced frequency for various rolling speeds. See legend Fig. 5a.

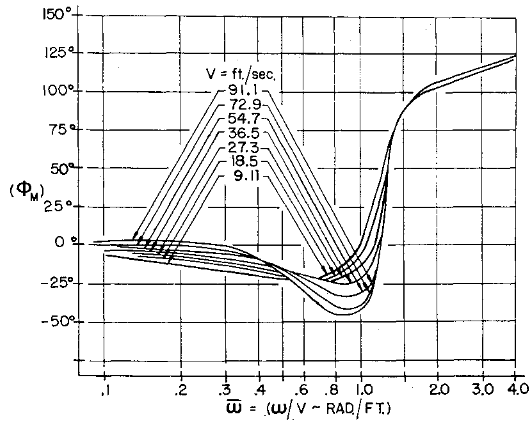

Fig. 5c Theoretical tire moment phase angle as a function of the reduced frequency for various rolling speeds. See legend Fig. 5a.

contact theory could be made. However, even when these parameters are held constant while the speed V is changed, Figs. 5 a and b show that the frequency response trends are quite similar to the experimental data for F, ϕ_F, and M.

The tire moment phase angle ϕ_M shown in Fig. 5c does not correlate too well with the experimental data of Fig. 4c. These results however are quite dependent on the values of d, k, K_ϕ which are the most difficult parameters to estimate due to lack of data. These parameters do not affect the F and M plots in Figs. 5a and 5b which do correlate well. Due to the difficulty of obtaining accurate experimental values of M and ϕ_M it is also quite likely that considerable error exists in Figs. 4b and 4c.

Generally it is noted in comparing Figs. 4 and 5 that rolling speed does affect the frequency response of tires and that the point contact theory can be used to predict the response at least as well as the stretched string theory although, at present insufficient experimental data is available for good quantitative studies in the high-frequency shimmy range. It should be noted from Refs. 2–4 that the frequency response predicted by the stretched string theories shows no variation of the quantities F, ϕ_F, M, ϕ_M with speed V. That is, they reduce the V parameter family of curves in each case to a single curve which does not correlate with the experimental data of Fig. 4. It should be noted also that Pacejka[3] discusses a tire inertia term which leads to a variation of the response with speed for M and ϕ_M, however the F, ϕ_F are unaffected. The empirical results of Ref. (8) are also obtained without regard to the rolling speed V.

V. Conclusions

The author realizes that more experimental evidence and further studies are necessary before the dynamics of the rolling, pneumatic tire can be fully understood however from the observations presented in this paper the following conclusions appear valid: 1) The point contact theory gives a valid mathematical representation of the tire for shimmy analysis over the entire range of natural shimmy frequencies when appropriate tire parameters are used. 2) When obtaining the frequency response of tires either theoretically or experimentally, the rolling speed V and the forced frequency ω should be considered separately, not simply the reduced frequency ω/v. 3) In frequency response analysis or experiment it is not necessary to consider forced frequencies above the ratio $\tilde{R} = 1$ (or $\omega h/v = 0.787$) as natural shimmy likely does not occur above this condition. 4) More experimental frequency response data on tire parameters is necessary if accurate theoretical shimmy analyses are to be made in the higher frequency range (structural-torsion shimmy mode). Otherwise it is necessary to analyze the landing gear system over large variations in several of the tire parameters in order to be certain that the stability of the system falls within the desired limits.

VI. References

[1] Collins, R. L., "Theories on the Mechanics of Tires and Their Application to Shimmy Analysis," *Journal of Aircraft*, Vol. 8, No. 4, April 1971, pp. 271–277.

[2] Segal, L., "Force and Moment Response of Pneumatic Tires to Lateral Motion Inputs," *ASME Journal of Engineering for Industry*, Feb. 1966.

[3] Pacejka, H., "The Wheel Shimmy Phenomenon," Ph.D. thesis, Dec., 1966, Delft Technical Institute, Delft, The Netherlands.

[4] Saito, Y., "A Study of Dynamic Steering Properties of Pneumatic Tires," *Proceedings of the Ninth International Automobile Technical Congress*, FISITA, 1962.

[5] Collins, R. L. and Black, R. J., "Tire Parameters for Landing Gear Shimmy Studies," *Journal of Aircraft*, Vol. 6, No. 3, May-June 1969, pp. 252–258.

[6] Ginn, J. L., et al., "The B. F. Goodrich Tire Dynamics Machine," Paper 490B, presented at SAE Meeting, National Automobile Week, Detroit, Mich., March 12–16, 1962.

[7] Smiley, R. F. and Horne, W. B., "Mechanical Properties of Pneumatic Tires with Special Reference to Modern Aircraft Tires," TR-R-64, 1960, NASA.

[8] Rogers, L. C. and Brewer, H. K., "Synthesis of Tire Equations for Use in Shimmy and Other Dynamic Studies," *Journal of Aircraft*, Vol. 8, No. 9, Sept. 1971, pp. 689–697.